Solving Problems with Labeled Numbers

When solving problems with "labeled" numbers (those with "units" such as grams or liters), the labels are treated in the same way as "P" or "y" in algebra. The problem

$$10 \text{ liters} \times \frac{2 \text{ grams}}{\text{liter}} = 20 \text{ grams}$$

is solved in the same way as

$$10\ \chi \times \frac{2\ y}{\chi} = 20\ y$$

In the problem above, the "units" or "label" of "liters" cancels since the same "unit" is in both the numerator and the denominator leaving the "unit" of grams in the answer.

Guide to Solving Problems with Labeled Numbers

â When performing calculations with labeled numbers, solve the problem with the numbers first, and then deal with the labels. For instance, in the problem

$$40\ m^2 \times \frac{25\ \text{blocks}}{2\ m^2} = 500\ \text{blocks}$$

Multiply and divide the numbers first

$$40 \times 25 \div 2 = 500$$

And then cancel the m^2 leaving the unit of blocks.

$$m^2 \times \frac{\text{blocks}}{m^2} = \text{blocks}$$

ã When adding or subtracting numbers which have labels, each number must have the same label for the quantities to be added or subtracted.

2 oranges + 4 oranges = 6 oranges

but

2 oranges + 4 apples = 2 oranges + 4 apples

ä When multiplying or dividing with labels, treat the labels as you would any other algebraic quantity. Labels can be multiplied or divided in the same way as "P" and "y" in algebra.

5 Newtons x 2 meters = 10 NewtonC meters

NOTE: In science, the "C" is used as a separator.

Likewise,

$$\frac{6\ \text{grams}}{3\ \text{liters}} = 2\ \frac{\text{grams}}{\text{liter}}$$

The answer in the above problem could also be written with a slash, "/," which means divide.

2 grams/liter or 2 g/R

å When dividing dimensionally described numbers by a labeled fraction, it is easier to invert the fraction and multiply.

$$\frac{36\ \text{grams}}{\dfrac{12\ \text{grams}}{\text{liter}}} = 36\ \text{grams}\ \text{x}\ \frac{1\ \text{liter}}{12\ \text{grams}} = 3.0\ \text{liters}$$

æ When squaring or cubing dimensionally described numbers, both the number portion and the label are squared or cubed.

$$(100 \text{ cm})^2$$

$$10{,}000 \text{ cm}^2 = 1 \text{ x } 10^4 \text{ cm}^2$$

Note: $\text{cm}^2 = (10^{-2})^2 \text{ m}^2 = 10^{-4} \text{ m}^2$

What would be $(20 \text{ mm})^3$?

$$20^3 \text{ x } (\text{mm})^3 = 8000 \text{ mm}^3$$

or $8000 (\text{ x } 10^{-3}\text{m})^3 = 8000 \text{ x } 10^{-9} \text{ m}^3$

$$8 \text{ x } 10^{-6} \text{ m}^3$$

Find the sum of 60. ounces + 5 ounces =

60. ounces + 5 ounces = 65 ounces

Calculate the product:

4.0 mm x 6.0 mm x 5.0 mm =

4.0 mm x 6.0 mm x 5.0 mm = 120 mm^3

Find the quotient in the problem

$$\frac{36 \text{ Newtons(N)} \times 3.0 \text{ meters(m)}}{72 \text{ seconds(s)}}$$

$$\frac{36 \text{ Newtons(N)} \times 3.0 \text{ meters(m)}}{72 \text{ seconds(s)}} = 1.5 \text{ N}\cdot\text{m/s}$$

What would you give as the answer in the problem:

$$6.0 \text{ km x } 2.0 \text{ km} = ?$$

$$6.0 \text{ km x } 2.0 \text{ km} = 12 \text{ km}^2$$

$$12\ (10^3\text{m})^2 = 12\ (10^3)^2\ \text{m}^2 = 1.2 \text{ x } 10^7\text{m}^2$$

Simplify the expression:

$$\frac{48 \text{ grams}}{50. \text{ liters}} \text{ x } \frac{5.00 \text{ liters}}{1.2 \text{ boxes}}$$

$$\frac{48 \text{ grams}}{50. \text{ liters}} \text{ x } \frac{5.00 \text{ liters}}{1.2 \text{ boxes}} = 4.0 \frac{\text{grams}}{\text{box}}$$

Calculate each of the following, express your answers with the correct dimensions, and with the correct number of significant digits.

â 25 Joules x 2.0 seconds = ________________

ã $\dfrac{64\ \text{Newton} \cdot \text{meters}}{\dfrac{16\ \text{Newtons}}{\text{meter}}} =$ ________________

ä $2.4\ \text{N/m}^3 \times 5.00\ \text{m}^3 =$ ________________

å $\dfrac{200\ \text{mol} \times 5\ \ell}{250\ °\text{C}} \times \dfrac{6\ °\text{C}}{12\ \ell} =$ ________________

DIMENSIONAL ANALYSIS

or

THE FACTOR-LABEL METHOD OF SOLVING PROBLEMS

The solution to problems frequently involves the process of *dimensional analysis*. *Dimensional analysis* is a systematic way of solving numerical problems by the conversion of units. The word *dimension* applies to the units or labels in which the quantities are measured or described. Typical units or labels are meters, grams, liters, etc.

Dimensional analysis involves using the "units", "dimensions", or "labels" of the number to determine the mathematical process (multiplication or division) needed to arrive at a solution to the problem.

Many students already solve problems using dimensional analysis intuitively without realizing it. For instance, how many inches are there in 6.0 feet?

$$6.0 \text{ feet} = ? \text{ inches}$$

Most students would probably logic that since there are 12 inches in one foot, then 6 x 12 = 72. Therefore, the answer is 72 inches.

This solution is actually a form of "dimensional analysis." In dimensional analysis, one studies the problem and finds the *conversion factor(s)* needed to change from one unit into the other. The student then multiplies by the *conversion factor*(s) in a way that produces an answer with the desired label.

In the problem above, the conversion factor is

1 foot = 12 inches

To solve the problem using dimensional analysis, you must multiply the known quantity (6 feet in this case) by a ratio of units from your conversion factor in a way that will produce an answer with the correct label.

By multiplying the 6.0 feet by the ratio, 12 inches over 1 foot, the feet cancel and the final unit becomes the desired inches.

$$6.0 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 72 \text{ inches}$$

What would be the conversion factor needed to solve the following problem?

100. pounds = ? kilograms

You would need a conversion between pounds and kilograms. The actual conversion factor is found on your list of English-metric conversions.

1 pound = 0.4536 kilogram

How would you solve the problem using dimensional analysis?

You would multiply 100. pounds by 0.4536 kilogram over 1 pound so that the pounds cancel and the answer has the unit of kilograms.

$$100 \text{ pounds} \times \frac{0.4536 \text{ kg}}{1 \text{ pound}} = 45.4 \text{ kg}$$

Some Helpful Hints for Solving Problems

â Those conversion factors which are definitions have an infinite number of significant digits.

In the conversion

3 feet = 1 yard

both the “3" and the “1" have an infinite number of significant digits.

In the conversion

1 mile = 1.609 km

the “1” has an infinite number of significant digits but the “1.609" has only 4 significant digits.

ã Metric-metric conversion factors can easily be created by substituting the exponent for the prefix in the metric measurement.

For example, to convert meters into millimeters, the exponential notation, 10^{-3}, is substituted for the prefix “milli.”

$$1 \text{ mm} = 1 \times 10^{-3} \text{ meter}$$

or by multiplying through the equation by 1000 (10^3), the relationship becomes

$$1000 \text{ mm} = 1 \text{ meter}$$

ä Square and cubic units can be created by squaring and cubing other units. For instance,

$$1 \text{ meter} = 100 \text{ cm}$$

Since both quantities are equal, both their squares and cubes are also equal.

$$(1 \text{ meter})^2 = (100 \text{ cm})^2$$

$$1 \text{ m}^2 = 10{,}000 \text{ cm}^2 = 1 \text{ x } 10^4 \text{ cm}^2$$

NOTE: $\text{cm}^2 = (10^{-2})^2 \text{ m}^2 = 10^{-4} \text{ m}^2$

likewise,

$$(1 \text{ meter})^3 = (100 \text{ cm})^3$$

$$1 \text{ m}^3 = 1{,}000{,}000 \text{ cm}^3 = 1 \text{ x } 10^6 \text{ cm}^3$$

Note: $\text{cm}^3 = (10^{-2})^3 \text{ m}^3 = 10^{-6} \text{ m}^3$

Convert 10 cm into _?_ mm.

Step â Find the conversion factor.

$$10 \text{ mm} = 1 \text{ cm}$$

Step ã Set up the problem by multiplying the original value by a ratio of the conversion factors such that the original unit cancels.

$$10 \text{ cm} \times \frac{10 \text{ mm}}{1 \text{ cm}}$$

Step ä Calculate the answer.

$$10 \text{ cm} \times \frac{10 \text{ mm}}{1 \text{ cm}} = 100 \text{ mm}$$

Convert 2.0 feet into __?__ centimeters

Step â Find the conversion factors.

1 foot = 12 inches

1 inch = 2.54 cm

Step ã Set up the problem by multiplying the original value by a ratio of the conversion factors such that the original unit cancels.

$$2.0 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

Step ä Calculate the answer.

$$2.0 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 61 \text{ cm}$$

Convert 60. miles/hour to ? meters/second

Step â Find the conversion factors.

1 mile = 1.609 km

1 km = 1000 m

1 hour = 3600 seconds

Step ã Set up the problem.

$$60. \frac{\text{mi}}{1 \text{ hr}} \times \frac{1.609 \text{ km}}{1 \text{ mi}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

Step ä Calculate the answer.

60. $\frac{\text{mi}}{1\ \text{hr}} \times \frac{1.609\ \text{km}}{1\ \text{mi}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{hr}}{3600\ \text{s}} = 27\ \text{m/s}$

Convert 5.0 yard^2 into _?_ inches^2

Step â Find the conversion factors.

$(1\ \text{yard})^2 = (3\ \text{ft})^2$ $1\ \text{yard}^2 = 9\ \text{ft}^2$

$(1\ \text{ft})^2 = (12\ \text{inches})^2$ $1\ \text{ft}^2 = 144\ \text{in}^2$

Step ã Set up the problem.

$$5.0\ \text{yd}^2 \times \frac{9\ \text{ft}^2}{1\ \text{yd}^2} \times \frac{144\ \text{in}^2}{1\ \text{ft}^2}$$

Step ä Calculate the answer.

$$5.0\ \text{yd}^2 \times \frac{9\ \text{ft}^2}{1\ \text{yd}^2} \times \frac{144\ \text{in}^2}{1\ \text{ft}^2} = 6500\ \text{in}^2$$

Convert 1.00 g/cm^3 into _?_ lb/ft^3

Step â Find the conversion factors.

1000 grams = 1 kg

1 pound = 0.4536 kg

$(1 \text{ in})^3 = (2.54 \text{ cm})^3$ $1 \text{ in}^3 = 16.4 \text{ cm}^3$

$(12 \text{ in})^3 = (1 \text{ ft})^3$ $1728 \text{ in}^3 = 1 \text{ ft}^3$

Step ã Set up the problem.

$$1.00 \ \frac{\text{g}}{\text{cm}^3} \ \text{x} \ \frac{1 \text{ kg}}{1000 \text{ g}} \ \text{x} \ \frac{1 \text{ lb}}{0.4536 \text{ kg}} \ \text{x} \ \frac{16.4 \text{ cm}^3}{1 \text{ in}^3} \ \text{x} \ \frac{1728 \text{ in}^3}{1 \text{ ft}^3}$$

Step ä Calculate the answer.

62.5 lb/ft^3

Calculate each of the following problems expressing your answer in the **exponential form**, with **one significant digit to the left of the decimal point**, with the correct **number of significant digits**, and correct **dimension**.

$(5.1 \times 10^{-4}\ \text{m})\ (3.0 \times 10^{11}\ \text{m}) =$ ______________

$$\frac{4.8000 \times 10^{-5}\ \text{g}}{1.60 \times 10^{-18}\ \text{g/ml}} =$$ ______________

$(7.53 \times 10^{11}\text{kJ}) + (3.22 \times 10^{10}\text{kJ}) =$ ______________

$(1.2 \times 10^{6}\ \text{mm})^2 =$ ______________

Calculate each of the following problems expressing your answer in the **exponential form**, with **one significant digit to the left of the decimal point**, with the correct **number of significant digits**, and correct **dimension**.

$(6.27 \times 10^{-5}\ \text{mg}) - (5.2 \times 10^{-7}\ \text{mg}) =$ ____________

$\sqrt[3]{6.4 \times 10^{-8}\,\text{pm}^3} =$ ____________

$(2.5 \times 10^{-9}\ \text{kg})(6.00 \times 10^{-6}\,\text{m/s}) =$ ____________

$\dfrac{7.5 \times 10^{11}\ \text{mol}}{2.50 \times 10^{23}\ \text{g/mol}} =$ ____________

Calculate each of the following problems expressing your answer in the **exponential form**, with **one significant digit to the left of the decimal point**, with the correct **number of significant digits**, and correct **dimension**.

$\sqrt[4]{6.25 \text{ x } 10^{26} \text{ cm}^8}$ = __________________

$(7.2 \text{ x } 10^{12} \text{ MW})(4 \text{ x } 10^{8} \text{ MW}) =$ __________________

$(3.00 \text{ x } 10^{-5} \text{ :m}^2)^3 =$ __________________

$\dfrac{5.5 \text{ x } 10^{6}\text{kg} + 5.0 \text{ x } 10^{5}\text{kg}}{1.2 \text{ x } 10^{-2} \text{ mol}} =$ __________________

Calculate each of the following problems expressing your answer in the **exponential form**, with **one significant digit to the left of the decimal point**, with the correct **number of significant digits**, and correct **dimension**.